

Uncertainty Estimation for Dense Prediction Tasks

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README.md

Uncertainty estimation models

We provide the implementation details of uncertainty estimation techniques. Including the ensemble based solutions and generative model based methods.



Code and tutorial material are available

0

Uncertainty is inherent within Machine Learning





Easy sample? Hard sample?

Outline

- Motivation
- Background
- Aleatoric uncertainty and Epistemic Uncertainty
- Uncertainty Approximation
 - ✓ Ensemble solutions
 - ✓ Generative model solutions
 - ✓ Bayesian latent variable model solutions
- Experiments
- Discussion and Conclusion

Distribution estimation vs Point estimation

Classification: prediction with confidence
Regression: prediction with variance

Uncertainty: a mechanism to understand model limitations

- Camouflaged object detection Classification
- Salient object detection -

• Which one is salient?



• Which one is salient?



• Where is the camouflaged object



Camouflaged Object Segmentation dataset: Anabranch Network for Camouflaged Object Segmentation, Le et al, CVIU, 2019

• Where is the camouflaged object



Motivation – Depth estimation from RGB

• What's the exact distance?



Motivation – Depth estimation from RGB

What's the exact distance?



 Visual ambiguity and thin items foreground, problems background (e.g., distance limitation of sensor, less data available)

- Model can make mistakes sometimes
- Model should be aware when it makes mistakes

Uncertainty: a mechanism to understand model limitations





Confidence of p(y|x)?

Variance of prediction?

Camouflaged object detection



Salient object detection

Monocular depth estimation

Figure 5: NYUv2 Depth results. From left: input image, ground truth, depth regression, aleatoric uncertainty, and epistemic uncertainty.

Figure from "What uncertainties do we need in bayesiandeep learning for computer vision?" by A. Kendall and Y. Gal.

- Well-calibrated model
- Hard-negative mining
- Dynamic confidence supervision

Confidence-Aware Learning for Camouflaged Object Detection, Liu, Zhang, Barnes, WACV 2022

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Given training dataset $D = \{x_i, y_i\}_{i=1}^N$, the goal of machine learning methods:

$$\min_{\theta} \mathbb{E}_{x,y}[\mathcal{L}(f(x,\theta),y)] = \int \mathcal{L}(f(x,\theta),y)dp(x,y) \approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}(f(x_i,\theta),y_i), (x_i,y_i) \sim p(x,y)$$

Loss over a continuous distribution is approximated by empirical loss over a dataset.

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$$\min_{\theta} \mathbb{E}_{x,y}[\mathcal{L}(f(x,\theta),y)] = \int \mathcal{L}(f(x,\theta),y)dp(x,y) \approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}(f(x_i,\theta),y_i), (x_i,y_i) \sim p(x,y)$$

 $\mathbb{E}_{x,y}$: Expectation over input $f(x,\theta)$: model hypothesis for x given model parameters θ \mathcal{L} : (.,.) loss $(x_i, y_i) \sim p(x, y)$: (x_i, y_i) input/output sampled from true distribution

Ambiguity comes from θ and the representativeness of the sampled dataset D.

Given training dataset $D = \{x_i, y_i\}_{i=1}^N$, the predictive distribution is defined as:

$$p(y|x) = \int p(y|x,\theta)p(\theta)d\theta$$

where the likelihood of prediction is defined as:

 $p(y|x,\theta) = \mathcal{N}(f(x,\theta),\Sigma)$ (Regression) Or $p(y|x,\theta) = Softmax(\frac{f(x,\theta)}{\exp(\sigma^2)})$ (Classification)

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(Classification)

Uncertainty origin: (ϵ / Σ), $p(\theta)$

Aleatoric uncertainty

(Intrinsic randomness of a phenomenon [1,2])

e.g., sensor noise, labeling noise

Epistemic uncertainty (Lack of knowledge of data [1,2]) Model noise

1. "Aleatory or epistemic? does it matter?, A. D. Kiureghian and O. Ditlevsen, Structural Safety, 2009.

2. What uncertainties do we need in Bayesian deep learning for computer vision? A. Kendall and Y. Gal. NeurIPS 2017

Uncertainty origin: (ϵ / Σ), $p(\theta)$

Aleatoric uncertainty: data related uncertainty (Intrinsic randomness of a phenomenon [1,2])

Epistemic uncertainty: model related uncertainty (Lack of knowledge of data [1,2])

> Point estimation system vs Self-awareness of machine learning system Decision making: Medical diagnosis, automatous driving, ...

1. "Aleatory or epistemic? does it matter?, A. D. Kiureghian and O. Ditlevsen, Structural Safety, 2009.

2. What uncertainties do we need in Bayesian deep learning for computer vision? A. Kendall and Y. Gal. NeurIPS 2017

Prediction: dog Probability: 0.98

Prediction: dog Probability: 0.95

Parsed an image of myself through the animal network and it's 98% confident I'm a dog.

Example and image from: <u>Dealing with Overconfidence in Neural Networks: Bayesian</u> <u>Approach – Jonathan Ramkissoon (jramkiss.github.io)</u>

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Aleatoric Uncertainty and Epistemic Uncertainty

- Aleatoric uncertainty: randomness, inherent noise
- Epistemic uncertainty: lack of knowledge, can be explained away with enough data, while aleatoric uncertainty cannot.

Aleatoric uncertainty: function of input, model it over outputs Epistemic uncertainty: function of model, model it over network parameters

Examples

What is aleatoric uncertainty? What is epistemic uncertainty?

Figure 1: **Illustrating the difference between aleatoric and epistemic uncertainty** for semantic segmentation on the CamVid dataset [8]. *Aleatoric* uncertainty captures noise inherent in the observations. In (d) our model exhibits increased aleatoric uncertainty on object boundaries and for objects far from the camera. *Epistemic* uncertainty accounts for our ignorance about which model generated our collected data. This is a notably different measure of uncertainty and in (e) our model exhibits increased epistemic uncertainty for semantically and visually challenging pixels. The bottom row shows a failure case of the segmentation model when the model fails to segment the footpath due to increased epistemic uncertainty, but not aleatoric uncertainty.

Figure from "What uncertainties do we need in bayesiandeep learning for computer vision?" by A. Kendall and Y. Gal.

Aleatoric uncertainty modeling

Image-level classification

Image-independent aleatoric uncertainty—homoscedastic uncertainty
Image-conditional aleatoric uncertainty---heteroscedastic uncertainty

Pixel-level Classification/Regression

Pixel-conditional uncertainty

Image-level uncertainty

Dataset-level uncertainty

Constant vs Learned

Aleatoric Uncertainty--Regression:

For Gaussian likelihood: $p(y|x,\theta) = \mathcal{N}(f(x,\theta), \Sigma(x))$

 $\Sigma(x) = diag((\sigma(x))^2)$ is the inherent label noise with

the basic assumption:

 $y = f(x,\theta) + n(x), n(x) \sim \mathcal{N}(0, \Sigma(x))$

Uncertainty-aware loss:
$$\mathcal{L}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{2(\sigma(x))^2} \mathcal{L}_2 + \frac{1}{2} \log((\sigma(x))^2) \right)$$

Aleatoric Uncertainty--Regression:

For Gaussian likelihood: $p(y|x,\theta) = \mathcal{N}(f(x,\theta), \Sigma(x))$

For numerical stability, define $s_i = \log((\sigma(x))^2)$: $\mathcal{L}(\theta) = \frac{1}{N} \sum_{i=1}^N \left(\frac{1}{2} \exp(-s_i) \mathcal{L}_2 + \frac{1}{2} s_i\right)$

Aleatoric uncertainty: $U_a = (\sigma(x))^2$

Aleatoric Uncertainty--Classification:

For SoftMax likelihood where $p(y|x,\theta) = Softmax(\frac{f(x,\theta)}{\exp(\sigma^2)})$

$$\mathcal{L}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{\exp(\sigma^2)} \mathcal{L}_{ce} + \frac{1}{2} \sigma^2 \right)$$

For numerical stability, define $T = \exp(\sigma^2)$ (the temperature):

$$\mathcal{L}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{T} \mathcal{L}_{ce} + \frac{1}{2} \log(T)\right) \qquad \qquad U_a = (\sigma(x))^2$$

Trivial solution:

For Gaussian likelihood: $p(y|x,\theta) = \mathcal{N}(f(x,\theta), \Sigma(x))$

 $\sigma(x)=1$!!, constant! Image-independent!!

Trivial solution:

For SoftMax likelihood where $p(y|x,\theta) = Softmax(\frac{f(x,\theta)}{\exp(\sigma^2)})$

$$\mathcal{L}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{\exp(\sigma^2)} \mathcal{L}_{ce} + \frac{1}{2} \sigma^2\right)$$

 $\sigma(x)=0$!! Temperature=1!!
Avoid the Trivial solution:

- Two types of aleatoric uncertainty definition
 Multi-head
 Mean entropy within BNN
- Uncertainty consistency loss for sampling free aleatoric uncertainty estimation

$$U_{a} = \mathbb{E}_{p(\theta|D)} [H(y|x,\theta)]^{L}$$

Stefan Depeweg et. al. Decomposition of Uncertainty in Bayesian Deep Learning for Efficient and Risksensitive Learning. ICML. 2018

Solve it:





Dual-head

Entropy

GT

Prediction

- Not directly estimated
- Defined as residual of predictive uncertainty and aleatoric uncertainty
- Mutual information of model prediction and model parameters within Bayesian Neural Network (BNN).

Given predictive uncertainty $U_p = H(y|x)$

Epistemic uncertainty: $U_e = U_p - U_a = H(y|x) - \mathbb{E}_{p(\theta|D)}[H(y|x,\theta)] = I(y,\theta|x)$

$$I(y,\theta|x) = H(y|x) - H(y|x,\theta) = H(y|x) - \mathbb{E}_{p(\theta|D)}[H(y|x,\theta)]$$

Stefan Depeweg et. al. Decomposition of Uncertainty in Bayesian Deep Learning for Efficient and Risksensitive Learning. ICML. 2018

For Gaussian likelihood with aleatoric uncertainty σ^2 from multi-head, the entropy based uncertainty is reduced to a function of variance, leading to epistemic uncertainty:

$$U_e = \mathbb{E}_{p(\theta|D)}[p(y|x,\theta)^2] - (\mathbb{E}_{p(\theta|D)}[p(y|x,\theta)])^2$$

The predictive uncertainty is then:

$$U_{p} = \mathbb{E}_{p(\theta|D)} [p(y|x,\theta)^{2}] - (\mathbb{E}_{p(\theta|D)} [p(y|x,\theta)])^{2} + \mathbb{E}_{p(\theta|D)} [\sigma(x)^{2}]$$

epistemic uncertainty aleatoric uncertainty

- What uncertainties do we need in Bayesian deep learning for computer vision? A. Kendall and Y. Gal. NeurIPS 2017
- 2. Stefan Depeweg et. al. Decomposition of Uncertainty in Bayesian Deep Learning for Efficient and Risksensitive Learning. ICML. 2018

SoftMax likelihood for classification:

 $U_p = H(y|x)$

$$U_e = U_p - U_a = H(y|x) - \mathbb{E}_{p(\theta|D)}[H(y|x,\theta)] = I(y,\theta|x)$$



Break

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- Aleatoric uncertainty and Epistemic Uncertainty

Uncertainty Approximation

- ✓ Ensemble solutions
- ✓ Generative model solutions
- ✓ Bayesian latent variable model solutions
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- Discussion and Conclusion

- Not directly estimated
- Defined as residual of predictive uncertainty and aleatoric uncertainty
- Mutual information of model prediction and model parameters within Bayesian Neural Network (BNN).

Given predictive uncertainty $U_p = H(y|x)$

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$$I(y,\theta|x) = H(y|x) - H(y|x,\theta) = H(y|x) - \mathbb{E}_{p(\theta|D)}[H(y|x,\theta)]$$

Stefan Depeweg et. al. Decomposition of Uncertainty in Bayesian Deep Learning for Efficient and Risksensitive Learning. ICML. 2018

Uncertainty Approximation

 Recall that epistemic uncertainty is usually directly estimated, which is defined as residual of predictive uncertainty and aleatoric uncertainty or the mutual information of model prediction and model parameters within Bayesian Neural Network (BNN).

Given predictive uncertainty $U_p = H(y|x)$

Epistemic uncertainty: $U_e = U_p - U_a = H(y|x) - \mathbb{E}_{p(\theta|D)}[H(y|x,\theta)] = I(y,\theta|x)$

$$I(y,\theta|x) = H(y|x) - H(y|x,\theta) = H(y|x) - \mathbb{E}_{p(\theta|D)}[H(y|x,\theta)]$$

Stefan Depeweg et. al. Decomposition of Uncertainty in Bayesian Deep Learning for Efficient and Risksensitive Learning. ICML. 2018

Uncertainty Approximation

- Recall: $U_e = U_p U_a = H(y|x) \mathbb{E}_{p(\theta|D)}[H(y|x,\theta)] = I(y,\theta|x)$
- no close form solution for $p(\theta|D)$

 $p(\theta|D) = p(\theta|x, y) = p(y|x, \theta)p(\theta)/p(y|x)$, where p(y|x) cannot be evaluated analytically, thus no close form solution for $p(\theta|D)$.

- Approximation for Bayesian posterior inference
 - Variational inference

Approximate $p(\theta|D)$ with easy-controlled distribution $q_{\gamma}(\theta)$, γ : variational parameters. i.e. MC-dropout

• Markov chain Monte Carlo (MCMC) methods

Sampling based solution, correlated sequence of $\theta_t \sim p(\theta|D)$. MC average is used as approximation of expectation----Generative model based solutions

Uncertainty Approximation

• Ensemble solutions

- 1. Dropout as a Bayesian Approximation: Representing Model Uncertainty in Deep Learning. Yarin Gal, Zoubin Ghahramani. ICML. 2016
- 2. Simple and Scalable Predictive Uncertainty Estimation using Deep Ensembles. Balaji Lakshminarayanan et. al. NeurIPS 2017

Generative model solutions

1. Adversarial distillation of Bayesian neural network posteriors. Kuan-Chieh Wang et. al. ICML. 2018

Bayesian latent variable model solutions

- 1. What Uncertainties Do We Need in Bayesian Deep Learning for Computer Vision? Alex Kendall, Yarin Gal. NeurIPS. 2017
- 2. Decomposition of Uncertainty in Bayesian Deep Learning for Efficient and Risk-sensitive Learning. Stefan Depeweg et. al. ICML. 2018

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 - ✓ MC-dropout
 - ✓ Deep ensemble
 - ✓ Snapshots ensemble
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• MC dropout:

True predictive distribution:

$$p(y|x) = \int p(y|x,\theta)p(\theta)d\theta$$

MC average as approximation: $p(y|x) \approx \frac{1}{T} \sum_{t=1}^{T} p(y_t|x,\theta_t)$

Where θ_t is sampled from the approximate posterior distribution $q_{\gamma}(\theta)$

Dropout as a Bayesian Approximation: Representing Model Uncertainty in Deep Learning. Yarin Gal, Zoubin Ghahramani. ICML. 2016

• MC dropout:

Implementation details: add dropout before every weighted layer during both training and testing.

Pros: easy to implement, no additional parameters Cons: cannot control the dropout mask, mode collapse issue

Masksembles for Uncertainty Estimation. Nikita Durasov et. al. CVPR. 2021.

• Deep ensemble



Simple and Scalable Predictive Uncertainty Estimation using Deep Ensembles. Lakshminarayanan, Balaji et. al. NeurIPS. 2017

• Deep ensemble

Implementation details: random initialization, multi-network, multi-head.

Pros: easy to implement, usually suffer no mode collapse issue Cons: extra parameters leading to longer training time, fixed number of predictions, not very flexible.

Snapshot ensemble



Snapshot Ensembles: Train 1, Get M for Free. Gao Huang et. al. ICLR. 2017

• Snapshot ensemble

Implementation details: save multiple snapshots for multiple predictions

Pros: no extra parameters, easy to implement Cons: hard to determine the snapshots point

Examples



MD: MC-dropout, DE: deep ensemble, SE: snapshot ensemble

Dense Uncertainty Estimation. Zhang et. al. 2021. https://github.com/JingZhang617/UncertaintyEstimatin

Ensemble based Uncertainty computation regression

- Aleatoric uncertainty $U_a = \frac{1}{T} \sum_{t=1}^{T} H(y|x, \theta_t)$, or $U_a = \sigma(x)^2$
- Epistemic uncertainty

$$U_e = \mathbb{E}_{p(\theta|D)}[p(y|x,\theta)^2] - (\mathbb{E}_{p(\theta|D)}[p(y|x,\theta)])^2$$

or
$$U_e = H\left(\frac{1}{T}\sum_{t=1}^T p(y|x,\theta_t)\right) - \frac{1}{T}\sum_{t=1}^T H(y|x,\theta_t)$$

• Predictive uncertainty

$$U_p = U_a + U_e$$

Ensemble based Uncertainty computation-classification

Aleatoric uncertainty

$$U_a = \frac{1}{T} \sum_{t=1}^{T} H(y|x, \theta_t)$$
, or $U_a = \sigma(x)^2$

• Predictive uncertainty

$$U_p = H(\frac{1}{T}\sum_{t=1}^{T} p(y|x,\theta_t))$$

• Epistemic uncertainty

$$U_e = H\left(\frac{1}{T}\sum_{t=1}^T p(y|x,\theta_t)\right) - \frac{1}{T}\sum_{t=1}^T H(y|x,\theta_t)$$

Examples

Aleatoric uncertainty: inherent noisy region Epistemic uncertainty: hard samples Predictive uncertainty: both



Dense Uncertainty Estimation. Zhang et. al. 2021. https://github.com/JingZhang617/UncertaintyEstimation

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Generative model solutions

- Latent variable models
- Energy-based model

Latent variable models

- A latent variable is a variable which is not directly observable and is assumed to affect the observable variables (manifest variables)
- A latent variable model is a statistical model that relates a set of observable variables to a set of latent variables (Latent Variable Models and Factor Analysis, David J. Bartholomew et. al, 2011)
- More formally, a latent variable model (LVM) is a probability distribution over two sets of variables x and z as p_θ(x, z), or over three sets of variables x, y and z as p_θ(y|x, z) for the conditional version of the latent variable models with the conditional variable x.



Latent variable model solutions

• Predictive distribution with extra latent variable z

$$p(y|x) = \int p_{\theta}(y|x,z)p(\theta)p(z)d\theta dz$$

Regression Likelihood:

$$p_{\theta}(y|x,z) = \mathcal{N}(f_{\theta}(x,z),\Sigma)$$

Uncertainty origin:
$$(\epsilon / \Sigma), z, p(\theta)$$

Aleatoric uncertainty

Classification Likelihood:

$$p_{\theta}(y|x,z) = Softmax(f_{\theta}(x,z)/\exp(\sigma^2))$$

Epistemic uncertainty

Latent variable model solutions

CVAE—Conditional Variational Auto-encoder

> Auto-Encoding Variational Bayes. Kingma, Diederik et. al. ICLR. 2014

Learning Structured Output Representation using Deep Conditional Generative Models. Sohn, Kihyuk et. al. NeurIPS. 2015

CGAN—Conditional Generative Adversarial Nets

➢ Generative Adversarial Nets. Goodfellow, Ian et. al. NeurIPS. 2014

Conditional Generative Adversarial Nets. Mehdi Mirza, Simon Osindero. arXiv. 2014

ABP—Alternating Back-Propagation

> Alternating Back-Propagation for Generator Network. Tian Han et. al. AAAI. 2016

Latent variable model solutions—VAE/CVAE

• VAE: unsupervised feature representation



• CVAE: latent feature exploring



• CVAE: conditional directed graph model



$$\mathcal{L}(\theta,\phi;x) = \mathbb{E}_{q_{\phi}(z|x,y)} \left[\log(p_{\theta}(y|x,z)) \right] - D_{KL}(q_{\phi}(z|x,y)||p_{\theta}(z|x))$$

- 1. A Probabilistic U-Net for Segmentation of Ambiguous Images. Simon Kohl et. al. NeurIPS. 2018
- 2. UC-Net: Uncertainty Inspired RGB-D Saliency Detection via Conditional Variational Autoencoders. Jing Zhang et. al. CVPR. 2020

Latent variable model solutions—GAN/CGAN

- GAN: min-max game
 - discriminator seeks to maximize the probability assigned to real and fake images

 $\max(\log(D(x)) + \log(1 - D(G(z)))) \text{ or } \min(\mathcal{L}_{ce}(D(x), 1) + \mathcal{L}_{ce}(D(G(z)), 0)))$

 generator learns to generate samples that have a low possibility of being fake by minimizing the log of the inverse probability predicted by the discriminator for fake images.

 $\min(\log(1 - D(G(z))))$

CGAN for dense prediction



 $\mathcal{L}_D = \mathcal{L}_{CE}(D(y), 1) + \mathcal{L}_{CE}(D(f_G(x, z), 0))$

Adversarial Learning for Semi-Supervised Semantic Segmentation. Wei-Chih Hung et. al. BMVC 2018

Explain ABP

- VAE—Inference model is needed
- GAN–Discriminator is needed with no inference model
- ABP—Alternating Back-Propagation

Infer the latent variable directly from the true posterior distribution
 No extra parameters

Alternating Back-Propagation

• Define the generative model as:

$$\begin{aligned} z &\sim p(z) = \mathcal{N}(0,1) \\ y &= f_{\theta}(x,z) + \epsilon, \epsilon \sim \mathcal{N}(0,\Sigma) \end{aligned}$$

• The conditional distribution of y is defined as:

$$p_{\theta}(y|x) = \int p(z)p_{\theta}(y|x,z)dz$$

• Define the observed data log-likelihood as $\log(p_{\theta}(y|x))$, it's gradient is then:

$$\frac{\partial}{\partial \theta} \log (p_{\theta}(y|x)) = \mathbb{E}_{p_{\theta}(z|x,y)} \left[\frac{\partial}{\partial \theta} \log (p_{\theta}(y,z|x)) \right]$$

Alternating Back-Propagation

• The observed data log-likelihood:

$$\frac{\partial}{\partial \theta} \log (p_{\theta}(y|x)) = \mathbb{E}_{p_{\theta}(z|x,y)} \left[\frac{\partial}{\partial \theta} \log (p_{\theta}(y,z|x)) \right]$$

where the expectation term can be approximated with Langevin dynamics based MCMC (a gradient based MCMC) to sample *z* from it's true posterior distribution via:

$$z_{t+1} = z_t + \frac{s^2}{2} \left[\frac{\partial}{\partial z} \log p_{\theta}(y, z_t | x) \right] + s \mathcal{N}(0, 1)$$

With:
$$\frac{\partial}{\partial z} \log p_{\theta}(y, z_t | x) = \frac{1}{\sigma^2} (y - f_{\theta}(x, z)) \frac{\partial}{\partial z} f_{\theta}(x, z) - z$$

t : time step for Langevin sampling *s* : step size

• ABP for dense prediction



Different from VAE or GAN that involves extra modules (inference model for VAE and discriminator for GAN), ABP sample directly from the true posterior distribution via gradient based MCMC.

Energy-based model

Latent variable model

estimate the distribution of the latent variableEstimate the predictive distribution

Energy-based model

Estimate predictive distribution directly.
Energy-based model solution

• EBM: energy-based model, learns an energy function to assign low energy to in-distribution samples and high energy for others.

• Energy-based model:

$$p_{\gamma}(y|x) = \frac{p_{\gamma}(y,x)}{\int p_{\gamma}(y,x)dy} = \frac{1}{Z(x;\gamma)} \exp\left[-U_{\gamma}(y,x)\right]$$

 $U_{\gamma}(y,x)$: the energy function

$$Z(x; \gamma) = \int \exp[-U_{\gamma}(y, x)] dy$$
: the normalizing constant

A tutorial on energy-based learning. Yann LeCun. 2006

Energy-based model solution

• Energy-based model: $p_{\gamma}(y|x) = \frac{p_{\gamma}(y,x)}{\int p_{\gamma}(y,x)dy} = \frac{1}{Z(x;\gamma)} \exp\left[-U_{\gamma}(y,x)\right]$

When the energy function U_{γ} is learned and input image x is given, prediction can be achieved via Langevin sampling: $y \sim p_{\gamma}(y|x)$:

$$y_{t+1} = y_t - \frac{\sigma^2}{2} \frac{\partial U_{\gamma}(y_t, x)}{\partial y} + \delta \Delta_t, \Delta_t \sim \mathcal{N}(0, 1)$$

Cooperative Training of Descriptor and Generator Networks. Xie, Jianwen et. al. TPAMI. 2018

Energy-based model solution

• EBM: 1) start point of Langevin sampling, 2) train the energy function U_{γ}

Start point:

- 1) Any deterministic model $f_{\theta}(x)$
- 2) Any latent variable model $f_{\theta}(x, z)$

Learn U_{γ} : maximum likelihood estimation

$$\Delta \gamma \approx \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial \gamma} U_{\gamma}(f_{\theta}(x_i), x_i) - \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial \gamma} U_{\gamma}(y_i, x_i)$$

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Bayesian latent variable model solutions

- Bayesian Neural Network
- Latent variable model
- Predictive distribution:

$$p(y|x) = \int p_{\theta}(y|x,z)p(\theta)p(z)d\theta dz$$

Uncertainty origin: $(\epsilon, \Sigma), z, p(\theta)$

Regression Likelihood:

$$p_{\theta}(y|x,z) = \mathcal{N}(f_{\theta}(x,z),\Sigma)$$

Aleatoric uncertainty

Epistemic uncertainty

Classification Likelihood:

 $p_{\theta}(y|x,z) = Softmax(f_{\theta}(x,z)/\exp(\sigma^2))$

Generative model based Uncertainty computation—regression

- Aleatoric uncertainty $U_a = \frac{1}{T} \sum_{t=1}^{T} H(y|x, \theta_t)$, or $U_a = \sigma(x)^2$
- Epistemic uncertainty

$$U_e = \mathbb{E}_{p(\theta|D)}[p(y|x,\theta)^2] - (\mathbb{E}_{p(\theta|D)}[p(y|x,\theta)])^2$$

or
$$U_e = H\left(\frac{1}{T}\sum_{t=1}^T p(y|x,\theta_t)\right) - \frac{1}{T}\sum_{t=1}^T H(y|x,\theta_t)$$

Predictive uncertainty

$$U_p = U_a + U_e$$

Generative model based Uncertainty computation--classification

• Aleatoric uncertainty

$$U_a = \frac{1}{T} \sum_{t=1}^{T} H(y|x, \theta_t)$$
, or $U_a = \sigma(x)^2$

• Predictive uncertainty

$$U_p = H(\frac{1}{T}\sum_{t=1}^{T} p(y|x,\theta_t))$$

• Epistemic uncertainty

$$U_e = H\left(\frac{1}{T}\sum_{t=1}^T p(y|x,\theta_t)\right) - \frac{1}{T}\sum_{t=1}^T H(y|x,\theta_t)$$

Break

Outline

- Motivation
- Background
- Aleatoric uncertainty and Epistemic Uncertainty
- Uncertainty Approximation
 - ✓ Ensemble solutions
 - ✓ Generative model solutions
 - ✓ Bayesian latent variable model solutions
- Experiments
- Discussion and Conclusion

Experiments--Uncertainty quality measure?

Expected calibration error

≻On Calibration of Modern Neural Networks. Chuan Guo et. al. ICML. 2017

• Patch accuracy vs patch uncertainty

Evaluating Bayesian Deep Learning Methods for Semantic Segmentation. Jishnu Mukhoti and Yarin Gal. Arxiv. 2018

• Evaluation on out-of-distribution samples

Experiments--Tasks

- Camouflaged object detection
- Salient object detection

COD

• Where is the camouflaged object



Camouflaged object detection

TABLE 1 Ensemble based solutions for **camouflaged object detection**. ↑ indicates the higher the score the better, and vice versa for ↓.

Method CAMO [82]		CHAME	ELEON [83]	COD10K [71]		NC4K [84]		
	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$
Base	.757	.079	.848	.029	.731	.035	.803	.048
MD	.767	.080	.842	.028	.731	.035	.803	.048
DE	.729	.088	.846	.030	.718	.037	.796	.051

1. Base: the base model

2. MD: MC-dropout

3. DE: deep ensemble

TABLE 3 Generative model based solutions for **camouflaged object detection**. ↑ indicates the higher the score the better, and vice versa for ↓.

Method	CAMO [82]		CHAMELEON [83]		COD10K [71]		NC4K [84]	
	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$
Base	.757	.079	.848	.029	.731	.035	.803	.048
CVAE	.758	.081	.848	.030	.731	.034	.802	.048
CGAN	.762	.080	.852	.026	.730	.034	.807	.048
ABP	.756	.081	.846	.030	.729	.034	.801	.047
EBM	.777	.076	.844	.031	.721	.038	.796	.050

- 1. Base: the base model
- 2. CVAE: the CVAE based framework
- 3. CGAN: the CGAN based framework
- 4. ABP: the ABP based framework
- 5. EBM: the EBM based framework

Predictive Uncertainty-Ensemble



Predictive Uncertainty-Generative Model



Aleatoric Uncertainty-Ensemble



Aleatoric Uncertainty-Generative Model



Epistemic Uncertainty-Ensemble



Epistemic Uncertainty-Generative Model



Three types of uncertainty



Observations

- Aleatoric uncertainty focus on inherent noise, i.e. object boundaries
- Epistemic uncertainty highlight challenging pixels, representing our ignorance about which model generated the given dataset.
- COD: aleatoric uncertainty vs epistemic uncertainty

SOD

• Which one is salient?



Salient object detection

TABLE 2 Ensemble based solutions for **salient object detection**. ↑ indicates the higher the score the better, and vice versa for ↓.

Method	DUTS [78]		DUT [79]		HKU-IS [80]		PASCAL [81]	
	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$
Base	.842	.037	.760	.055	.904	.030	.828	.064
MD	.854	.036	.763	.056	.911	.028	.840	.061
DE	.828	.040	.738	.061	.897	.031	.825	.065

- 1. Base: the base model
- 2. MD: MC-dropout
- 3. DE: deep ensemble

TABLE 4

Generative model based solutions for **salient object detection**, \uparrow indicates the higher the score the better, and vice versa for \downarrow .

Method	DUTS [79]		DUT [80]		HKU-IS [81]		PASCAL [82]	
	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$	$F_{\beta}\uparrow$	$\mathcal{M}\downarrow$
Base	.842	.037	.760	.055	.904	.030	.828	.064
CVAE	.836	.037	.748	.055	.901	.030	.826	.063
CGAN	.846	.035	.752	.054	.905	.029	.828	.063
ABP	.829	.040	.740	.059	.889	.034	.818	.068
EBM	.834	.040	.744	.062	.900	.031	.829	.064

- 1. Base: the base model
- 2. CVAE: the CVAE based framework
- 3. CGAN: the CGAN based framework
- 4. ABP: the ABP based framework
- 5. EBM: the EBM based framework

Predictive Uncertainty-Ensemble



Predictive Uncertainty-Generative Model



Aleatoric Uncertainty-Ensemble



Aleatoric Uncertainty-Generative Model



Epistemic Uncertainty-Ensemble



Epistemic Uncertainty-Generative Model



Three types of uncertainty



MC-dropout

CGAN

Observations

- Aleatoric uncertainty focus on inherent noise, i.e. object boundaries
- Epistemic uncertainty highlight challenging pixels, representing our ignorance about which model generated the given dataset.
- SOD: aleatoric uncertainty vs epistemic uncertainty

Discussion

- Sampling-free uncertainty estimation
- Pixel-level uncertainty vs Instance-level uncertainty
- How to effectively use the produced uncertainty
- Model calibration and uncertainty estimation
- Out-of-distribution detection and uncertainty estimation
- Multi-modal/multi-task learning
- Semi-/weakly-supervised learning
- Effectiveness measure

Thanks Bushfire CoE for the support.



Contact: zjnwpu@gmail.com



Code and tutorial material are available

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